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Disjunctive Numerals of Estimation

1. Introduction

English contains a number of expressions of the form *m or n*, where *m* and *n* are numerals, with the meaning 'from *m* to *n*.' I call such expressions disjunctive numerals of estimation, or DNEs. Examples 1–4 illustrate the use of these expressions:

(1) My family's been waiting four or five hours for a flight to Florianopolis.
(2) This saying has fifteen or twenty meanings.
(3) Let me have thirty or forty dollars.
(4) Your call will be answered in the next ten or twenty minutes.

Example 1 may be used to assert that my family has been waiting from 4 to 5 hours, not for either exactly 4 or exactly 5 hours. More precisely, one should say that the expression is ambiguous between, on the one hand, an exact or literal interpretation of *four or five*, in which my family is said to have been waiting either 4 or 5 hours and not some intermediate amount of time (such as 4 hours and 30 minutes) and, on the other hand, an idiomatic "estimation" interpretation of *four or five*, in which my family is said to have been waiting from 4 to 5 hours; and one should also say that the second interpretation is much more likely to be given to the sentence than the first. The range interpretation of DNEs is even clearer in examples 2–4, where the interval between *m* and *n* is greater than 1. For example, sentence 2 under this interpretation is true if the number of meanings of the saying in question has any value from 15 to 20 and is false otherwise, similarly for examples 3 and 4.

The literal and idiomatic interpretations of a DNE coincide in truth value if the interval between *m* and *n* equals 1 and the DNE modifies a noun that is normally enumerated in discrete (integral) values, as in 5:

(5) There are four or five people ahead of us in line.

In such cases, nevertheless, the literal and idiomatic interpretations of DNEs should be distinguished. Under the latter, 5 is understood as providing an estimate of the number of people ahead of us in line, whereas under the former, it is understood as specifying a disjunction of exactly how many people are ahead of us in line.

DNEs have been around for a long time in English. The following excerpt is from a report written shortly after the British takeover of New Amsterdam in 1664, quoted in Keller. DNEs are italicized:

(6) There are about *nine or ten* three Mast Vessels of about *eighty or a Hundred* tons burthen, *two or three* ketches and Barks of about forty Tun, and about twenty Sloops of about *twenty or five and twenty* Tun belonging to the Government—all of which Trade for England, Holland and the West Indies, except *five or six*, sloops that use the river Trade to Albany and that way. (34)

Not all expressions of the form *m or n*, qualify as DNEs, however, as the following examples show:

(7) My family's been waiting two or five hours for a flight to Florianopolis.
(8) This saying has eleven or twenty-two meanings.
(9) Let me have forty or thirty dollars.

Example 7 would be used to say only that my family has been waiting either 2 or 5 hours, and not for an intermediate period of time, such as 3 hours. Thus the expression *two or five* in that example behaves only as an ordinary disjunction, not as a DNE. Similarly, example 8 would be used to say that the saying in question has either 11 or 22 meanings, not some intermediate number, such as 16. Finally, example 9 would be used to request either $40 or $30, not any amount of money in between.

2. Principles of Interpretation for DNEs

Genuine DNEs in English conform to certain principles of formation. For convenience in stating these principles, let M and N be the numbers that the numerals m and n respectively stand for; let D be the difference between N and M; and let R be the ratio of M to N. The principles are given in 10–12.[1]

(10) D is a nonnegative power of 10 or is 1/2 or 1/5 of a nonnegative power of 10 (i.e., is the result of dividing a nonnegative power of 10 by one of its prime factors).

(11) For any given value of D, the DNEs can be arranged in a series such that the M-value of each successive DNE is identical to the N-value of its predecessor, and D is a divisor of the M- and N-values of each DNE in the series.

(12) $R < 1$ (or $M < N$).

The expression *two or five* in example 7 violates principle 10, because the difference between 2 and 5 (viz. 3) is neither a power of 10 nor the result of dividing a power of 10 by one of its prime factors. The expression *thirty or fifty* in example 8 violates principle 11, because the difference between 21 and 11 (viz. 10),

although a power of 10, is not a divisor of either 11 or 21.[2] Finally, the expression *forty or thirty* in example 9 violates principle 12, because 40 (M) is greater than 30 (N). In example 13, I display some of the DNE series consistent with principles 10–12:

(13) *D* Series
   a. ½   one half or one,[3] one or one and a half, one and a half or two, two or two and a half, . . .
   b. 1   one or two, two or three, three or four, . . .
   c. 2   six or eight, eight or ten, ten or twelve, . . .
   d. 5   five or ten, ten or fifteen, fifteen or twenty, . . .
   e. 10   ten or twenty, twenty or thirty, thirty or forty, . . .
   f. 20   sixty or eighty, eighty or a hundred, . . .
   g. 50   fifty or a hundred, a hundred or a hundred fifty, a hundred fifty or two hundred, . . .
   h. 100   a hundred or two hundred, two hundred or three hundred, three hundred or four hundred, . . .

I name each of these DNE series by its *D*-value; thus, I call series 13a the ½-series, 13b the 1-series, and so on. The series in which *D* is 1/5 of a power of 10 are defective. First, there is no series in which $D = 1/5$. Second, the 2-series apparently lacks the members in which $M < 6$, and in which $N > 12$. Third, the 20-series lacks the members in which $M < 60$, and in which $N > 100$.[4] Fourth, no larger DNE series exists in which *D* is 1/5 of a power of 10. As *R* approaches unity, expressions in the series I have listed (13) are less likely to be used as DNEs. Thus, for example, the expression *nine hundred ninety-five or a thousand* is much less likely to occur as a DNE (or for that matter as an ordinary disjunctive expression) than, say, *fifteen or twenty*. Although that is not a sufficient reason to reject such expressions as potential DNEs,[5] whenever it is possible to "factor" disjunctive numerical expressions, as in *two or three hundred* and *twenty-five or -six* (as in the title of the Chicago Transit Authority song "Twenty-Five or -Six to Four"), it is preferable to do so.

   The full use of DNEs is possible only in unlimited numerical domains. For example, 1-series DNEs (13b) can be used to express approximate "clock" time, which is limited to the numerals from *one* to *twelve*, but not 5-series DNEs (13d). Thus while 14a is ambiguous between a literal and DNE interpretation, 14b is unambiguous, having a literal interpretation only. However, the circular property of clock time (i.e., that one o'clock follows twelve o'clock) legitimates the DNE interpretation of the expression *twelve or one (o'clock)* as in 14c.

   (14) a. The ceremony starts at nine or ten (o'clock).
         b. The ceremony starts five or ten (o'clock).
         c. The ceremony starts at twelve or one (o'clock).

3. Other DNE Series

Each of the DNE series in 13 can be thought of as a "decimal" series, that is, a series in which the difference between the smaller and larger number in each member is a power of 10 or a factor of a power of 10. The defective 2- and 20-series, however, may be analyzed differently, as residues of a duodecimal (based on 12) and of a vigesimal (based on 20) series, respectively. The 2-series may be analyzed as a duodecimal series, since 2 is an integral submultiple of 12 (the result of dividing 12 by 6). Analyzing it as such may help explain why the series begins with a DNE in which M is half of 12 and ends with a DNE in which $N = 12$. That the 20-series may be analyzed as a vigesimal series is obvious, since 20 is the first power of 20. The numbers that are used in the defective 20-series correspond to the beginning, middle, and endpoints of the counting by twenties that holds in modern standard French (*soixante*, ..., *soixante-dix-neuf*, *quatre vingts*, ..., *quatre-vingts-dix-neuf*, *cent*), which Karl Menninger has argued is the residue of an ancient vigesimal system that entered the French language in the eleventh century. If these reanalyses of the 2- and 20-series are correct, then principle 10 can be revised as in 15:

(15) D is a nonnegative power of 10 or is 1/2 of a nonnegative power of 10.

4. Parasitic DNEs

Certain DNEs that do not conform to principles 10 and 11 are nevertheless well formed since they correspond to DNEs that do conform using a different unit of measure, as in the following examples:

(16)   a. Your shipment will go out in sixty or ninety days.
       b. Your shipment will go out in two or three months.

(17)   a. They need eight or twelve more quarts of water for that recipe.
       b. They need two or three more gallons of water for that recipe.

(18)   a. I'll get there in seven or ten days' time.
       b. I'll get there in one or one and a half weeks' time.

The DNEs in 16a, 17a, and 18a may be called "parasitic" because their well-formedness depends on their approximate or exact equivalence (together with their units of measure) with their counterparts in 16b, 17b, and 18b. Only a few parasitic DNEs out of the many that can be constructed appear to be acceptable.

5. Syntactic Conditions on DNEs

To be understood as DNEs, the disjunction must be of numerals and not of larger phrases containing numerals. Thus examples 19–21 are understood quite differently from their counterparts 1–3.

(19)   My family's been waiting four hours or five hours for a flight to Florianopolis.
(20)   This saying has fifteen meanings or twenty meanings.

(21) Let me have thirty dollars or forty dollars.

In example 19, the wait is described as lasting either 4 hours or 5 hours; the interpretations of 20–21 are similar. As a consequence of this restriction, disjunctions involving the frequency words *once* and *twice* cannot be considered to be DNEs. To see this, consider the following examples:

(22) a. Ana has played that sonata one or two times.
      b. Ana has played that sonata one time or two times.
      c. Ana has played that sonata once or twice.

(23) a. Ana has played that sonata two or three times.
      b. Ana has played that sonata two times or three times.
      c. Ana has played that sonata twice or three times.

In 22a and 23a, numerals alone are conjoined, with the result that they can be construed as DNEs. In 22b and 23b, entire nonnumeral phrases are disjoined, with the result that the expressions cannot be construed as DNEs, just as in the case of the disjunctive phrases in examples19–21. Now consider 22c and 23c. In 22c, the words *once* and *twice* are disjoined, but each is understood as a nonnumeral phrase; hence the disjunction as a whole cannot be construed a DNE.[6] Example 23c is even more compelling, since in it the word *twice* is disjoined with the entire nonnumeral phrase *three times* and hence must be understood as of the same syntactic type as the latter. Again, the conclusion follows that the disjunction as a whole cannot be a DNE.

6. DNEs Containing the Indefinite Article

When a noun is modified by the indefinite article *a* or *an*, expressing unity, it may be postmodified by the phrase *or two*. The entire construction is equivalent to the DNE *one or two*, as in the following examples:

(24) a. You'll have to wait an hour or two.
      b. You'll have to wait one or two hours.

Since the expression *an . . . or two* is a DNE, the subphrase *or two* is in construction with the indefinite article *an*, and not with the phrase *an hour*, since DNEs must be disjunctions of numerals, not of nonnumeral phrases. Thus the construction of *or two* in 24a contrasts with that of *or so* in 25, which is in construction with the entire phrase *an hour*.

(25) You'll have to wait an hour or so.

The difference in construction between the phrases *or two* and *or so* becomes apparent when these are associated with phrases in which an indefinite article modifies a numeral, such as *hundred*, or *thousand*, as in the following examples:

(26) a. These changes took place over a period of a hundred or two years.
      b. These changes took place over a period of a hundred or so years.

    c. ?These changes took place over a period of a hundred years or two.
    d. These changes took place over a period of a hundred years or so.

In 26a, the DNE *a . . . or two* modifies the numeral *hundred*, and the interpretation of the phrase headed by *years* is 'from one hundred to two hundred years.' On the other hand, in 26b, the disjunctive expression *a hundred or so* modifies *years* and the phrase as a whole has the interpretation 'roughly one hundred years.' In 26c, the construction *a hundred years or two* is syntactically odd, and even if well formed, it does not give rise to a DNE, since the interpretation would be 'one hundred years or two years.'[7] Finally, the expression *a hundred years or so* in 26d is perfectly normal; the phrase *or so* is disjoined to *a hundred years*, and the interpretation is identical to that of *a hundred or so years* in 26b.

    Just as the phrase *or two* must be postposed to a noun when unity is expressed by the indefinite article, so must the phrase *and a half*, as the following examples illustrate:

    (27) a. This manuscript will take a year and a half to edit.
          b. This manuscript will take one and a half years to edit.

The obligatory postposing of phrases coordinated with the indefinite article is a consequence of the fact that the indefinite article cannot be an immediate constituent of a coordinate compound. Accordingly, the DNE that is equivalent to *one and a half or two* must have both the phrases *and a half* and *or two* postposed when it modifies a noun and unity is expressed by the indefinite article, as in the following example:

    (28) ??I'll be with you in a minute and a half or two.

In 28, the phrase *and a half* is intended to be in construction with *a*, and the phrase *or two* is, in turn, intended to be in construction with the discontinuous phrase *a . . . and a half*. For many, if not most, speakers of English, however, sentence 28 is ungrammatical under this interpretation.[8] For such speakers, apparently, at most one phrase can be in extraposition with the indefinite article.

    The indefinite article also cannot appear without a phrase for it to modify and is a proclitic on whatever word follows it. Consequently, the counterparts to the DNEs *one half or one* and *one or one and a half* that modify a noun and that express unity by the indefinite article must repeat the noun in both members of the disjunction, as in the following examples:

    (29) a. Please cut a half inch or inch off the top.
          b. Please cut half an inch or an inch off the top.
          c. Please cut a half inch or an inch off the top.

    (30) a. Grind a kilo or kilo and a half of farinha for him.
          b. Grind a kilo or a kilo and a half of farinha for him.

These examples can be interpreted as containing DNEs, despite the fact that nonnumeral expressions are disjoined (cf. section 5 above).[9]

7. Ordinal DNEs

Ordinal numerals, as well as cardinal numerals, can be disjoined to form DNEs, as in this sentence:

(31)   She should be finishing up the fifteenth or twentieth chapter by now.

However, the range covered by ordinal DNEs must be integral since there are no ordinal numerals corresponding to the fractional numerals *one and a half*, and so on; hence there is no ordinal series corresponding to the ½-series in 13a. Ordinal numerals (with the word *half* obligatorily replacing the word *second* and with *quarter* optionally replacing *fourth*) are also used to express the denominators of fractions in English, and thus can enter into DNEs under certain conditions. When the numerator is unity, the fractions as a whole or just the denominators can be disjoined to form a DNE, as long as the denominators belong to a legitimate DNE series, as in the following examples:

(32)   When I lived in Pernambuco, I spent only one-fifth or (one)-sixth of my salary on rent.
(33)   You should use only a fifteenth or (a) twentieth of that solution when you mix the plant spray.

When the numerator has any other value, then the result is difficult if not impossible to interpret as a DNE, whether or not the numerator is repeated, as in the following example:

(34)   You should use only three-fifteenths or (three)-twentieths of that solution when you mix the plant spray.

However, different numerators can be used if $M = K/(K+1)$ and $N = (K+1)/(K+2)$, where $0 < K < 3$, as in the following examples:[10]

(35)   I used to flunk a half or two-thirds of my students every semester.

(36)   Now I find myself spending two-thirds or three-quarters of my time doing mindless paperwork.

The DNEs in 35 and 36 are both well formed despite the fact that the value of $1/M$ is greater than that of $1/N$. For some speakers, if the denominators form a 1-series and are relatively small in value, the fractions may be reversed (so that the numerically smaller fraction is put first) and the result is still a DNE, as in the following example:

(37)   When I lived in Cambridge, I spent a third or a half of my salary on rent.

There is another DNE series of fractional numerals based on successive division by 2, as follows:

(38)   . . . , an eighth or (a) quarter/fourth, a quarter/fourth or (a) half

This series conforms to principle 39:

    (39) *N* is a negative power of 2, and *M* is the next larger (in absolute value) power of 2.

The following are examples of the use of DNEs from the series in example 38:

    (40) Be sure to add a quarter or a half teaspoon of salt to the batter.
    (41) Please take an eighth or a quarter inch off the sides.

How far this series extends depends on the speaker and the domain in which DNEs from it are being used. The expression *a quarter* (or *fourth*) *or a half* is in general use in all domains. In the domain of measuring things in inches, the series may extend, depending on the speaker and audience, as far as *a sixty-fourth or a thirty-second*. In this domain, also, the numerals may be reversed, so that the smaller (in absolute value) negative power of 2 comes first, as in the following example, which is equivalent in meaning to example 41:

    (42) Please take a quarter or an eighth inch off the sides.

Finally, when the denominator is fixed, an ordinary DNE can be used as the numerator, as in the following examples:

    (43) a. There are one or two thirds of a minute left to play.
           b. The reaction time was thirty or forty hundredths of a second.
           c. Two or three quarters of the audience has already left.

The DNE *a half or three quarters* can be considered parasitic on the DNE in 43c.

8. Analysis

The special interpretation of DNEs is not the result of a conversational implicature, since the interpretation is not cancelable by failure to observe the cooperative principle. Nor does it appear to be the result of a conventional implicature, since if such an implicature could be motivated for one expression of the form *m or n*, it is motivated for all of them, not just the ones that happen to conform to the principles 10–12 and 39. Rather, it appears that DNEs are idiomatic phrases of some sort, and the question arises as to why there are so many of them, since the kinds of idiomatic phrases that are usually analyzed, such as *bury the hatchet*, are sui generis. We are not used to analyzing whole collections of idiomatic phrases that systematically vary in form and meaning. There is, however, one familiar class of phrasal idioms in English that do vary in this way, and that is the class of numerical phrases that express numbers greater than 99.

      Consider, for example, the phrase *two hundred and ninety-five*. This expression can be construed either as the coordination of two smaller numerals (*two hundred*, *ninety-five*) or as a single numeral which happens to be conjunctive in form. The former interpretation we consider literal, and the latter idiomatic. In some sentences, such as 44a, the expression receives only the latter, idiomatic

interpretation; in others, such as 44b, it receives only the former, literal interpretation; and in still others, like 44c, it can receive both.

(44) a. Ana's score was two hundred and ninety-five.
      b. Ana's scores on the two tests were two hundred and ninety-five, respectively.
      c. Ana's scores were two hundred and ninety-five.

Not all conjunctions of two or more numerals in English can be construed idiomatically as a single compound numeral; to be so construed, they must conform to well-known laws analogous to the principles that govern the form of DNEs (Hurford; Greenberg). For example, while *two hundred and ninety-five* is a well-formed numeral phrase in English, *ninety-five and two hundred* is not. Thus the existence of DNEs and the restrictions on their form in English, though apparently unique to that language, are not all that surprising. Like numeral phrases, they are a subset of a much larger set of phrases that are of regular syntactic form and that receive special interpretations that can be specified by relatively simple and, in some cases, general principles. These principles are based on properties of the decimal system (and perhaps also to a limited extent the duodecimal and vigesimal systems) of counting, and on successive division by 2 in the case of principle 39.

Notes

I began the work reported on here in the late 1970s and read a version of it at the 1978 NYSCOL meeting in Albany. It was inspired in part by Háj Ross's work on coordinate constructions, especially on "me first." I am delighted to let it see the light of day in this volume.

[1] Channell gives examples illustrating 10 and 11, and states 12 explicitly. Her commentary on the examples she uses to illustrate these principles is simply that "there are some constraints on what the two numbers [*M* and *N*] may be" (466–67). I thank Adrienne Lehrer for bringing Channell's paper to my attention.

[2] However, John R. Stafford, when chairman and chief executive officer of American Home Products Corporation, was quoted as saying that the merger of his company with A. H. Robins Company would place his company in the "top three or five" drug companies in the world if the courts were to approve the acquisition plan (*New York Times* 15 Feb. 1988: D1). No one I have talked to on the subject recognizes *three or five* as a DNE, but apparently Stafford did use it as such on this occasion, since it is very unlikely that he knew that the result of his company's merger with Robins would place it in either the top three or the top five drug companies in the world, but not in the top four. Perhaps he meant to say "top three to five," or was misquoted.

[3] The expression *one half or one* is somewhat awkward as a DNE. However, there are alternative, more idiomatic ways of expressing this range as a DNE; see section 6.

[4] According to Channell, the upper bound (largest value for *N*) for the 2-series is 20, not 12; and the upper bound for the 20-series is 160 (467).

[5] Channell gives 20 as the upper bound for the 1-series, 100 as the upper bound for the 5-series, 130 (!) as the upper bound for the 10-series, and 900 as the upper bound for the 50-series.

[6] The phrase *once or twice* in 22c, like *one or two* in 22a, may be construed ironically to indicate that Ana has played the sonata in question many times, but this interpretation is irrelevant to its analysis as an DNE.

[7] The postnominal coordination of an adjective in a construction with a premodifying adjective, as in *sober men and true* is a figure of speech in English; if numerals are used, the result is interpreted as a DNE only if the figure is equivalent to a phrase containing a DNE, such as *fifteen minutes or twenty*.

[8] One person I asked about this example offered the interpretation 'one or one and a half minutes' for the phrase *a minute and a half or two*, claiming that *or two* in this example is short for *or two halves* and that therefore *or* is connecting *a minute and a half* with *two halves*, the latter meaning 'two half minutes,' or 'one minute'!

[9] The expression *a week or ten days* can be considered a partially parasitized DNE based on *a week or a week and a half*.

[10] In these examples, both the numerator and the denominator are part of a 1-series, and the value of the first disjunct is less than that of the second. The series appears to be defective, since examples in which *K* is large are not intuitively felt to be DNEs. Exactly at what point the series ends, however, is not clear.

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